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On ‘Two girls – the value of information’: Michael Jewess writes: The concluding paragraph of the interesting and stimulating article [1] is a little dismissive of the following statement in *Wikipedia*: ‘The answer [to the two-girls problem] depends on how this information comes to us – what kind of selection process brought us this knowledge.’ But *Wikipedia* has a point, which complements the article, and which can be demonstrated by means of a game show.

In a game show, both the host and the contestant are mathematically skilful, and each plays to win. In accordance with the game rules, the game show host first chooses at random a family with two children, and conceals them from the contestant. The host has access to full information about the children, but initially the contestant knows only their number (two) and the fact of random selection. The host then provides some further information about the children to the contestant. Next, in the light of the further information, the contestant computes the probability x that the family contains two girls, and thereby the probability $(1 - x)$ that it does not. Finally, the contestant guesses whether or not the family has in fact two girls (in line with the higher of the computed probabilities so as to maximise his chance of receiving the prize offered for a correct guess).

Problem 3 as worked out in the article corresponds to such a game in which (i) the rules additionally prescribe that the host provides the further information only in response to a challenge as follows –

Challenge A: By selecting one option within the square brackets, construct a true statement from ‘[At least one/neither] of the two children is a girl who was born on a Tuesday.’ –

and in which (ii) the response is in fact –

Response R: ‘At least one of the two children is a girl who was born on a Tuesday.’

On receiving Response R, the contestant computes, as in the article by reference to its Table 1, that $x = 13/27$.

But Table 1 also shows that, for a randomly-chosen family with two children, there is only a $27/196$ probability that Response R is a truthful response to Challenge A. If Response R is untrue (a $169/196$ probability), the host must give the following response:

Response S: ‘Neither of the two children is a girl who was born on a Tuesday.’

A contestant receiving Response S computes from Table 1 that $x = 36/169$. (Check: $\frac{27}{196} \times \frac{13}{27} + \frac{169}{196} \times \frac{36}{169} = \frac{1}{4}$, the probability of two girls in a randomly-chosen family with two children.)

Suppose instead that the challenge in the game is the following:

Challenge B: By selecting one option within each pair of square brackets, construct a true statement from ‘At least one of the two children is a [girl/boy] who was born on a [Sunday/Monday/Tuesday/Wednesday/Thursday/Friday/Saturday].’

To Challenge A there is only one truthful response for any particular family in question. But to Challenge B there exist either one or two truthful responses depending on the family in question. If there are two truthful responses, and the one is less helpful to the contestant than the other, then the host will give the less helpful one.

If there are two boys ($1/4$ probability in a randomly-chosen family with two children), the host is forced to respond to Challenge B in a way that admits the existence of a boy – and to identify the birth day of one of the two boys. The contestant then knows, regardless of what birth day is given, that $x = 0$ and the contestant will win the game for certain. If there are two girls ($1/4$ probability in a randomly-chosen family with two children) or a boy and a girl ($2/4$ probability in a randomly-chosen family with two children), the host opts for ‘girl’ over ‘boy’, and identifies the birth day of one of the two girls or of the girl respectively. Regardless of whether the host's response is identical with Response R or whether it is Response R with ‘Tuesday’ substituted by a different day, the contestant then computes that $x = 1/(1 + 2) = 1/3$. (Check: $\frac{1}{4} \times 0 + \left(\frac{1}{4} + \frac{2}{4}\right) \times \frac{1}{3} = \frac{1}{4}$.)

Judged as games, neither the game with Challenge A nor that with Challenge B is very interesting: whatever response the host gives, the contestant's better guess is always that the family does not contain two girls – though, interestingly for the mathematician, only just (by $1/27$), if Response R is given to Challenge A.

The above demonstrates that what one deduces from Response R depends on what the challenge was. A similar issue arose in a real TV game show, Monty Hall's *Let's make a deal*. In the Monty Hall game, the host knows which one (and only one) of three doors hides a prize, and the contestant has to guess which one. Marilyn vos Savant [2] imagines a little green woman who arrives from a UFO part-way through the game, in time to observe that the host has opened one door showing there is no prize there, but not in time to know the challenge to which the host has responded. Conceivably, the challenge to the host might have been: ‘Open as you choose any one of the three doors;’ or ‘Open a particular door which I the contestant choose;’ or – the actual, rather interesting case – ‘Open as you choose either one of a particular pair of doors which I the contestant choose.’ Unaware of what form the challenge has taken and of what choices (if any) the contestant has made, the little green woman is less able than the contestant to guess correctly where the prize is. The context in which the information has been supplied is crucial knowledge.

References

1. Keith Parramore and Joan Stephens, Two girls – the value of information, *Math. Gaz.*, **98** (July 2014), pp. 243-249.
2. Marilyn vos Savant, quoted by Paul Hoffman, *The man who loved only numbers* (Hyperion; Fourth Estate) 1998, p. 236.